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A Systematic Approach to Comparative Decision-Making Using Pseudo inverses

WHITE PAPER

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Abstract

There exist a number of circumstances in businesses where we make comparative decisions. Whether it be a set of ideas, concepts, or people, what we essentially assume is that there exists some "potential" associated with each one of them, and we assign values to those potentials by comparing them with one another. This is a very difficult problem if the number of “substances under comparison” is large. Here it is beneficial to split the problem into small, simpler problems and solve them all at once. It is also beneficial to get a quantitative measure of our understanding about the problem we are solving. Here, in this paper, we show how to take the bigger problem, split it into smaller problems, form a set of linear equations and solve these equations using standard techniques from linear algebra. The method here also evaluates our understanding about the problem and the requirement for a thorough evaluation.

In essence, this whitepaper provides a systematic approach to effectively compare options by reducing the subjectivity and noise which hinders effective “present-day decision making.”

Abbreviations

Sl. No.	Acronyms	Full form
1	DAR	Design Analysis and Resolution
2	Inv	Moore Penrose Inverse
3	LA	Linear Algebra
4	CMD	Care for Minor Details
5	SUC	Substances under comparison

Current Challenges

- Large set of ideas to compare
- Person to person variations
- Noise
- Increased time to delivery

Market Trends/Challenges

In any given scenario, we have multiple options and we have to make a choice based on our needs. As an engineer, for any problem or need, we develop multiple options with each having its own advantages and disadvantages. The challenge faced here is how to select the best option and what is the rationale for selecting a particular option.

The industry constantly faces this challenge with regard to infrastructure, technology, people, etc., and often relies on past experience to make a decision. This brings a lot of subjectivity into the decision, and may not be the best solution for the situation.

Currently, some of the widely-used decision making tools are Pugh matrix, Delphi technique, decision tree method, nominal group technique and cost effect analysis. Out of these techniques -- Pugh matrix -- is one of the best methods which is widely used.

The above-mentioned methods have some or all of the following limitations:

- High level of noise
- High level of subjectivity
- Lack of enough input to reduce subjectivity
- Require large number of iterations, as assigning of the potentials to the SUCs is ad hoc.
- Longer brainstorming time
- Increased time to delivery
- Thus inability to cope up with market requirements
- Doesn't give any idea about the understanding of the problem

Solution methodology

- Define the problem
- Split the problem
- Solve it at once

Solution and a Case Study

The core concept:

The method focuses on splitting the problem into smaller problems and solving them mathematically. It also quantifies the understanding of the problem, that is, the vagueness associated with the problem. This method tries to minimize the subjectivity and noise in the problem solving and gives a more quantitative result.

Key features of the method:

- Asks for more input

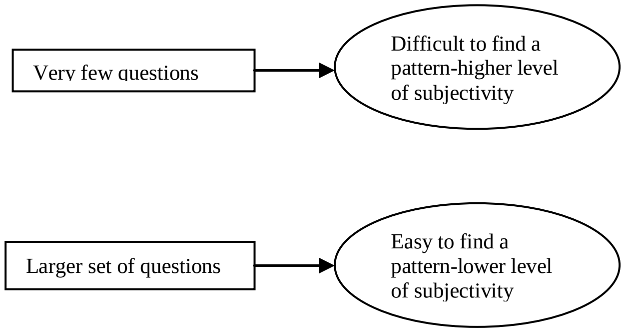

Figure 1: Flow chart to execute the process

- Gives the best possible solution for the given input
- Quantifies the understanding of the problem
- Alerts about the need for further study

A flow chart to illustrate the methodology is shown below.

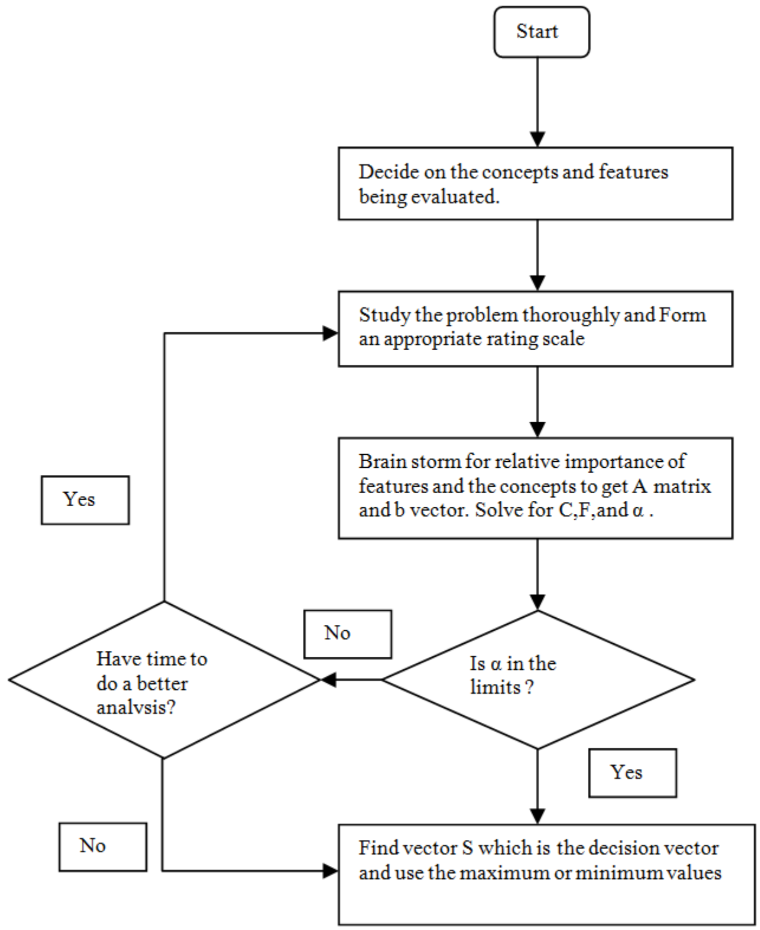

Figure 2: Flow chart to execute the process

A mathematical formulation and a case study:

For illustration, let's take a case where we need to select the best concepts from a set of four concepts (concept1, concept2, concept3 and concept4). Let there be four features (feature1, feature2, feature3 and feature4) with which we evaluate them.

Now we need to split the problem into smaller problems.

Bigger problem: Which concept holds the highest potential with respect to all features?

Smaller problems:

- Does concept1/feature1 or concept 2/feature2 have a higher potential with respect to (w.r.t.) the.feature1/overall objective:

If it's concept1 (if concept2 is bigger, take the negative of the assigned values)

Is it: a) Very different (assigned value 5)

b) Somewhat different (assigned value 3)

c) Almost the same (assigned value 2)

d) Same (assigned value 0)

This question needs to be asked for all the different concepts and features.

Where ratings 5, 3, 2, 0, -2, -3, 5 are purely arbitrary and could be modified on a case-by-case basis to improve efficiency.

Let F_i-F_j=b_{ij} for every i> j

= M*F=b

Where F represents "potentials" of the features, M the difference matrix and bk the assigned differences. b could be a discretized function in the fully automated case with relevant information about the function.

The M matrix for a four feature and four concept is obtained as

M=[1,-1,0,0;1,0,-1,0;1,0,0,-1;0,1,-1,0;0,1,0,-1;0,0,1,-1;0,0,0,1]

A brainstorming was conducted using the above-mentioned rating scale, and a vector b obtained in this special case is
b=[-2,3,3,5,2,2,3]'

Solving these linear equations using Moor Penrose inverse, we get
F=Inv(M)*b [where F is the vector giving the potentials of the different features]

Applying a similar methodology to the concepts, we get:
K*C_1=a_1 [c_1- c_N are the vectors that give the potential of different SUC with respect to different features and operating with K gives the difference in potentials of each SUC with respect to each feature]
K*C_2=a_2
....
K*C_N=a_N
In matrix form, we get:
K*$\mathbf{C}$=**A**

where C has $c_1,c_2,...c_N$ as its column vectors and A has $a_1,a_2...a_N$ as its column vectors.
Solving for C using the pseudo inverses, we get:
C=Inv(**K**)*__A__

In this special case
K=[1,-1,0,0;1,0,-1,0;1,0,0,-1;0,1,-1,0;0,1,0,-1;0,0,1,-1;0,0,0,1]
and A is obtained by brainstorming.

The table used for comparing the concepts during a brainstorming session is given below. The compared value between i th concept with the j th one w.r.t. different features is termed as $C_{ij.}$ The value of the final concept is assigned a particular value to avoid singularity which doesn't change the relative differences in the solution.

	w.r.t feature one(a_1)	w.r.t feature two(a_2)	w.r.t feature three(a_3)	w.r.t feature four(a_4)
C_{12}	-5	-5	-2	-2
C_{13}	3	5	5	5
C_{14}	2	2	3	3
C_{23}	0	5	2	0
C_{24}	2	0	5	5
C_{34}	3	-2	2	3
C_4	5	5	5	5

Table 1: The table used for brainstorming

Thus matrix A is obtained as:

A=[-5,3,2,0,2,3,5;-5,5,2,5,0,-2,5;-2,5,3,2,5,2,5;-2,5,3,0,5,3,5]

The "congruency error" which shows the how congruent the inputs are could be estimated by the formulation given below. This procedure helps in giving us an idea as to how correctly we understand the problem.

E=(A-KC)'*(A-KC)
where the diagonal numbers of the matrix **E** give the incongruence error with respect to each feature; the other elements show how each error depends the other error terms.

Then set all non-diagonal elements of the **E** matrix to zero forming a matrix **D** as the focus is only on the error with respect to each feature and interdependencies between those errors are not considered meaningful.

Let **magA**=[mag(a1)^-1, mag(a2)^-1,.....mag(aN)^-1]
sinA=**D*** magA'

sinA will be a vector showing the sine of the angle that the input vector subtends with the column space.

Angle = α = arcsine(sinA)

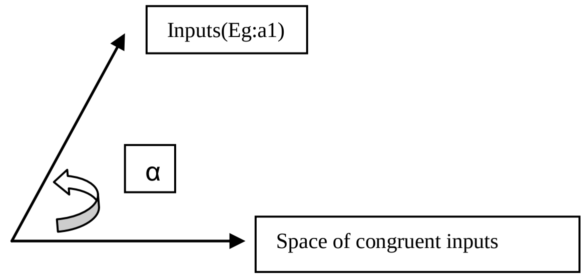

Figure 3: Angle subtended by the inputs to the space of congruent inputs

The diagram above shows exactly what angle the inputs are subtending with respect to the column space of the K matrix where α is a measure of conflicts in the inputs given or the vagueness of the problem. This error can be corrected in two ways -- first by understanding the problem more correctly and redoing the exercise or changing the rating scale to a better scale and redoing from the beginning.

S=C*F [Dot product of each column vector with the F vector gives the weighted average of the Scores (S) obtained by each concepts]
Where S represents the final scores obtained by each of the concepts.
Maximum or minimum value S would be the result depending on the situation.

The solutions for this special case are graphed below.

Future Work

- Adding a reliability matrix
- Developing software to implement this algorithm

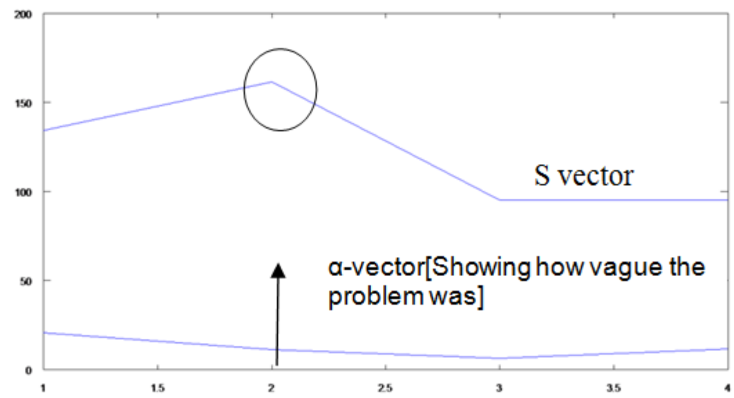

Figure 4: S and α are plotted w.r.t. concepts and features respectively. It could be noted that the second concept is the best out of them and maximum incongruence of inputs is w.r.t the feature one

The α value w.r.t the first feature is the highest [20 degrees]which shows the first feature was having the maximum level of conflicts in the inputs. After a further analysis with a changed ratings and a thorough understanding of the feature, one could reduce the value of α which is not done in this case as this is only for illustrative purposes. A better understanding of the problem and a good rating scale will reduce the α value near to zero. The appropriate minimum value of α could be set depending upon the problem and the time constraints.

The result two has been accepted as the best possible result in this case.

Future Work

There could also be situations where the confidence level of the equations varies with one another. There are situations where the equations are coupled. In those circumstances, we could add a confidence matrix R along with this set of equations, taking care of these situations. R matrix will be a diagonal matrix in the case where the equations are not coupled; otherwise it should be a full matrix. Software could also be developed to implement this algorithm.

Common Issues

The DAR technique which is currently used faces a set of problems. It takes a lot of time to assign the potentials, as this process is

Advantages

- Systematic and robust
- Reduced subjectivity and noise
- Information on vagueness of the problem understanding
- Reduced delivery time

iterative and highly subjective. The problem, if large, is that it's impossible to assign numbers mentally, which makes it subjective and slow.

The solution provided here, though, addresses these issues to a great extent, and automation is extremely important for the method to work efficiently. It is also clear that nothing can replace human judgment. So that expertise in the domain is imperative to make use of this technique effectively.

Conclusion

It could be concluded that in comparison to the current methods, the new approach gives a more robust and intuitive result. The following points try to substantiate the key points.

- Pugh matrix, as a method, asks for only very few inputs, so the magnitude of the impact in the problem design is affected heavily by individual inputs. These inputs can vary drastically from person to person, and thus the decision. But the current method tries to minimize the subjectivity by asking for *more inputs*, thus *reducing subjective variations* assuming that for a larger set of questions, the answers from different experts having similar experience in the field shall be more or less the same, or along similar lines
- In the current methods, while we try changing a particular ranking for a set of concepts, there needs to be a corresponding change made in other values of the concepts which we can't bring in, and thus causes noise in the problem. Use of pseudo inverses take care of this problem by finding the best possible solution.
- Comparative evaluation helps doing a thorough evaluation of the problem as the required inputs are the differences in potentials between them
- The incongruences or the amount of conflicting inputs are measured by an alpha term introduced here, and this will help quantify the understanding we have of the problem.
- Reduces the time in overall decision making
- More robust, intuitive, and easy to automate result

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